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#### 14. ABSTRACT

The main objective of the proposal concerns the development, analysis, implementation, and application of efficient and accurate numerical methods for interface problems, and problems on irregular domains, with applications in computational fluid dynamics and control of incompressible flows modeled by Navier-Stokes equations. Under the support of the current ARO grant, we have made significant progress for interface problems for the time-dependent incompressible flows, the two-phase flows based on the immersed interface method.

#### 15. SUBJECT TERMS

interface problems, irregular domains, Navier-Stokes equations, fluid-structure interactions, open and traction boundary conditions, free boundary probelm

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UU	UU	υυ	UU		19b. TELEPHONE NUMBER 919-515-3210

#### **Report Title**

Final Report: Simulation, control, and applications for flow and scattering problems

#### **ABSTRACT**

The main objective of the proposal concerns the development, analysis, implementation, and application of efficient and accurate numerical methods for interface problems, and problems on irregular domains, with applications in computational fluid dynamics and control of incompressible flows modeled by Navier-Stokes equations. Under the support of the current ARO grant, we have made significant progress for interface problems for the time-dependent incompressible flows, the two-phase flows based on the immersed interface method.

We believe that we have succeeded in completing all the proposed projects in the proposal. The research results also lead to new research directions and projects that are of interests of ARO.

# Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

### (a) Papers published in peer-reviewed journals (N/A for none)

Received		<u>Paper</u>
03/19/2010	1.00	X. Yang, X. Zhang, Z. Li, G. He. A smoothing technique for discrete delta functions with application to immersed boundary method in moving boundary simulations, Journal of Computational Physics, (03 2010): . doi:
03/19/2010	4.00	Z. Li, M. Lai, G. He, H. Zhao. An augmented method for free boundary problems with moving contact lines, Computers and Fluids, (01 2010): . doi:
03/19/2010	3.00	Y. Gong, Z. Li. Immersed Interface Finite Element Methods for Elasticity Interface Problems with Non-Homogeneous Jump Conditions, Numer. Math. Theor. Meth. Appl., (02 2010): . doi:
03/19/2010	2.00	F. Wang, J. Chen, W. Xu, Z. Li. An additive Schwarz preconditioner for the mortar-type rotated Q1 FEM for elliptic problems with discontinuous coefficients, Applied Numerical Mathematics, (03 2010): . doi:
07/09/2011	5.00	CHIN-TIEN WU, ZHILIN LI, AND MING-CHIH LAI. ADAPTIVE MESH REFINEMENT FOR ELLIPTIC INTERFACE PROBLEMS USING THE NON-CONFORMING IMMERSED FINITE ELEMENT METHOD, INTERNATIONAL JOURNAL OF $\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$
07/09/2011	8.00	Z. Li and MC. Lai. New Finite Difference Methods Based on IIM for Inextensible Interfaces in Incompressible Flows, East Asian Journal on Applied Mathematics, (05 2011): . doi:
07/09/2011	7.00	J. Xu, Z. Li, J. Lowengrub and H. Zhao. Numerical Study of Surfactant-Laden Drop-Drop Interactions, Commun. Comput. Phys., (08 2011): . doi:
07/09/2011	6.00	X. FENG, Z. LI AND Z. QIAO. HIGH ORDER COMPACT FINITE DIFFERENCE SCHEMES FOR THE HELMHOLTZ EQUATION WITH DISCONTINUOUS, Journal of Computational Mathematics, (02 2011): . doi:
08/10/2012	12.00	Zhilin Li, Peng Song. An Adaptive Mesh Refinement Strategy for ImmersedBoundary/Interface Methods, COMMUNICATIONS IN COMPUTATIONAL PHYSICS, (08 2012): 515. doi:
08/10/2012	16.00	Juan Ruiz Álvarez, Jinru Chen, Zhilin Li. The IIM in polar coordinates and its application to electro capacitance tomography problems, Applied Numerical Mathematics, (04 2011): 405. doi:

08/10/2012 14.00 Jian Hao, Zhilin Li, Sharon R. Lubkin. An augmented immersed interface method for moving structures with mass,
Discrete and Continuous Dynamical Systems - Series B, (06 2012): 1175. doi:

Discrete and Continuous Dynamical Systems - Series B, (06 2012): 1155. doi:

or with interfaces,

08/10/2012 15.00 Zhilin Li, Xiaohai Wang. Some new finite difference methods for Helmholtz equations on irregular domains

- 08/10/2012 13.00 Songming Hou, Zhilin Li, Liqun Wang, Wei Wang. A NumericalMethod for Solving Elasticity Equationswith Interfaces, COMMUNICATIONS IN COMPUTATIONAL PHYSICS, (08 2012): 595. doi:
- 08/22/2012 18.00 Kazufumi Ito, Bangti Jin. A new approach to nonlinear constrained Tikhonov regularization, Inverse Problems, (10 2011): 0. doi: 10.1088/0266-5611/27/10/105005
- 08/22/2012 19.00 Kazufumi Ito, Zhilin Li, and Zhonghua Qiao. The Sensitivity Analysis for the Flow Past ObstaclesProblem with Respect to the Reynolds Number,
  Advances in Applied Mathematics and Mechanics, (02 2012): 0. doi:
- 08/22/2012 17.00 Bangti Jin, Kazufumi Ito, Jun Zou. A direct sampling method to an inverse medium scattering problem, Inverse Problems, (02 2012): 0. doi: 10.1088/0266-5611/28/2/025003
- 08/28/2013 20.00 Ji-Chuan Liu, Kazufumi Ito. Recovery of inclusions in 2D and 3D domains for Poisson's equation, Inverse Problems, (07 2013): 0. doi: 10.1088/0266-5611/29/7/075005
- 08/30/2013 25.00 I. Caraus and Z. Li. Numerical Solutions of the System of Singular Integro-Differential Equations in Classical Holder Spaces,
  Advances in Applied Mathematics and Mechanics, (12 2012): 737. doi:
- 09/04/2011 9.00 Kazufumi Ito, Karim Ramdani, Marius Tucsnak. A time reversal based algorithm for solving initial data inverse problems,
  Discrete and Continuous Dynamical Systems Series S, (06 2011): 641. doi:
- 09/20/2014 27.00 C. H. Wang, J. Wang, Q. Cai, Zhilin Li, H. Zhao, R. Luo. Exploring accurate Poisson-Boltzmann methods for biomolecular simulations,

  Computationa and theoretical Chemestry, (11 2013): 34. doi:
- 09/20/2014 28.00 Quanxiang Wang, Zhiyue Zhang, Zhilin Li. AFourierfinitevolumeelementmethodforsolvingtwo-dimensionalquasi-geostrophicequationsonasphere,
  Applied Numerical Mathematics, (09 2013): 1. doi:
- 09/20/2014 29.00 W. M. Botello-Smith, XP. Liu, Q. Cai, Zhilin Li, H. Zhao, R. Luo. Numerical Poisson–Boltzmann model for continuum membrane systems,

  Chemical Physics Letters, (01 2013): 274. doi:
- 11/26/2014 30.00 Kazufumi Ito, Karl Kunisch. Optimal Control with \$L^p(\Omega)\$, \$p\in [0,1)\$, Control Cost, SIAM Journal on Control and Optimization, (04 2014): 0. doi: 10.1137/120896529
- 11/26/2014 31.00 Kazufumi Ito, Karl Kunisch. A variational approach to sparsity optimization based on Lagrange multiplier theory,
  Inverse Problems, (01 2014): 0. doi: 10.1088/0266-5611/30/1/015001
- 11/26/2014 33.00 Kazufumi Ito, Tomoya Takeuchi. Immersed Interface CIP for One DimensionalHyperbolic Equations, Commun. Comput. Phys., (07 2014): 96. doi:

TOTAL: 25

# (b) Papers published in non-peer-reviewed journals (N/A for none)

Received	<u>Paper</u>
08/28/2013 22.00	Kazufumi Ito , Bangti Jin , Jun Zou . A two-stage method for inverse medium scattering, Journal of Computational Physics, (02 2013): 211. doi:
08/30/2013 23.00	. 1. Adaptive mesh refinement techniques for the immersed interface method applied to flow problems, Computer structure, (04 2013): 249. doi:
08/30/2013 24.00	P. Song, J. , Xue, & Z. Li, . 2. Simulation of longitudinal exposure data with variance-covariance structures based on mixed models, , RIsk Analysis, (03 2013): 469. doi:
09/20/2014 26.00	Jianjun Xu, Yunqing Huang, Ming-Chih Lai, Zhilin Li. A coupled immersed interface and level set method for three-dimensional interfacial flows with insoluble surfactant, COMMUNICATIONS IN COMPUTATIONAL PHYSICS, (08 2012): 0. doi:
TOTAL:	4
Number of Papers	published in non peer-reviewed journals:
	(c) Presentations
Number of Present	eations: 4.00
	Non Peer-Reviewed Conference Proceeding publications (other than abstracts):
Received	<u>Paper</u>
TOTAL:	

Number of Non-Fe	Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):  Peer-Reviewed Conference Proceeding publications (other than abstracts):		
Received	<u>Paper</u>		
TOTAL:			
Number of Peer-R	eviewed Conference Proceeding publications (other than abstracts):		
	(d) Manuscripts		
Received	<u>Paper</u>		
08/22/2011 10.00	Kazufumi Ito, Karl Kunisch. Semismooth Newton Methods for Time-Optimal Control for a Class of ODEs , SIAM Journal of Control and Optimization (08 2011)		
08/22/2011 11.00	Kazufumi Ito, Tomoya Takeuchi. CIP immersed interface methods for hyperbolic equations withdiscontinuous coefficients,		
	SIAM Journal on Numerical Analyses (08 2011)		

Books

**Number of Manuscripts:** 

<u>Book</u>

Received

TOTAL:

08/28/2013 21.00 Kazufumi Ito. Nonsmooth Optimization Method and Sparsity, Basel: Springer Basel, (03 2013)

TOTAL: 1

#### **Patents Submitted**

#### **Patents Awarded**

#### **Awards**

#### **Graduate Students**

NAME	PERCENT_SUPPORTED	Discipline
Guangyu Chen	0.10	
Xiaomei Zhu	0.10	
FTE Equivalent:	0.20	
Total Number:	2	

## **Names of Post Doctorates**

<u>NAME</u>	PERCENT_SUPPORTED	
FTE Equivalent:		
Total Number:		

## **Names of Faculty Supported**

l	<u>NAME</u>	PERCENT_SUPPORTED	National Academy Member	
1	Zhilin Li	0.40		
١	Kazufumi Ito	0.40		
١	FTE Equivalent:	0.80		
	Total Number:	2		

# Names of Under Graduate students supported

<u>NAME</u>	PERCENT_SUPPORTED		
FTE Equivalent: Total Number:			
This section only appli	Student Metrics es to graduating undergraduates supported by this agreement in this reporting period		
	ber of undergraduates funded by this agreement who graduated during this period: 0.00 duates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields: 0.00		
•	ates funded by your agreement who graduated during this period and will continue raduate or Ph.D. degree in science, mathematics, engineering, or technology fields: 0.00		
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00  Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00			
The number of undergraduat	The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00		
	The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00		
	Names of Personnel receiving masters degrees		
<u>NAME</u>			
Total Number:			
	Names of personnel receiving PHDs		
<u>NAME</u> Elgaddafi Elamami <b>Total Number:</b>	1		
	Names of other research staff		
NAME	PERCENT_SUPPORTED		
FTE Equivalent: Total Number:			

**Sub Contractors (DD882)** 

**Inventions (DD882)** 

#### **Scientific Progress**

Our specific aims involved the development of accurate sharp interface methods based on Cartesian grids and immersed interface treatment for incompressible flows with a free boundary or a moving interface.

One of the most remarkable progresses is that we have developed an efficient augmented IIM for modeling the fluid structure interactions between a fluid flow modeled by the Stokes equations and a porous media modeled by the Darcy's law. There are many applications of the coupled Stokes and Darcy flow, for example, the underground water flow, oil recovery, fluid flow through tissues and cells. Our new idea is to introduce several augmented variables along the interface between the fluid flow and the porous media so that the problem can be decoupled as several Poisson equations. The augmented variables are chosen so that the Beavers-Joseph-Saffman (BJS) and other interface conditions are satisfied. In the discretization, the augmented variables have co-dimension one compared with the primitive variables and are solved through the Schur complement system. Our work is the first using Cartesian meshes and fast Poisson solvers fur such a coupling. The method can easily apply to other coupled systems and applications.

Working with Dr. Jianlin Xia of Purdue University, we have developed an efficient preconditioning technique for the Schur complement system arises from the augmented IIM. The preconditioning techniques have shown to be very efficient for all the application problems that we have tested including the flow past obstacles, contact line problems; drop spreading, and the couple of the Stokes flow and Darcy's law. Equipped with the new preconditioning technique, the Augmented IIM has become a direct challenger of the traditional boundary integral method.

Furthermore, we continue our research in the immersed finite element method (IFEM). We have developed the Robin-Robin domain decomposition method for Stokes-Darcy coupling using modified Cartesian meshes. Based on Z. Li's early non-conforming IFE method, we have developed a symmetric and consistent IFE method which is second order accurate for elliptic interface problems with homogeneous and non-homogeneous jump conditions. This is a significant step forward in the IFEM research. The paper has been accepted by the Journal of Scientific Computing. Our rigorous proof of the immersed boundary is first order accurate in the infinity norm for elliptic interface problems has been accepted by the journal of Mathematics of Computations. This may be a 40-year unsolved problem.

We have developed the direct sampling method for inverse medium problems,

specifically determine distribution of scatters by near field scattering data

from a single plane incident. We developed the probing index function over the medium domain that provides an estimate of the distribution of scatters.

We use the fundamental solution to Helmholtz equations, i.e., the probing

index uses the inner product of scattering data with the fundamental solution

at the measurement boundary. The effectiveness of the probing function is tested for several simulated data. We also extend the probing method for

the EM inverse scattering problems based on Maxwell equations.

We have developed a comprehensive Lagrange multiplier theory for

a general class of nonsmooth and non-convex optimizations including Non-Newtonian mechanics and sparsity optimizations. Nonsmooth solution method

is increasingly important to construct the sharp and enhanced solutions for scientific computations in general. Based on the theory we developed a semi-smooth Newton method in the form of primal dual active set method. We have demonstrated the effectiveness and applicability of our proposed algorithm for

tomography problems and control problems and nonlinear least square solution for reconstruction of medium coefficients.

We have developed the weak formulation for the fluid-structure interaction

based on the incompressible Navier Stokes equations. The formulation provides a systematic and accurate model for the iteration. Based on the formulation

we introduced an energy preserving weak solution and we developed the numerical simulation method for a general class of the fluid-structure interaction problems. We have a very promising preliminary result and more numerical tests are currently carried out.

**Technology Transfer**